ANNALS OF HUMAN BIOLOGY, 2001, VOL. 28, NO. 4, 395–402

Statistical characterization of the pubertal growth spurt

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Received 26 June 2000; accepted 27 September 2000

Summary. *Background*: This is a methodical investigation into the problem of estimating parameters for the pubertal spurt (PS). The variability involved in determining the timing, intensity and duration of the PS for height, leg height and biiliac width is estimated via a realistic simulation. Further, a decomposition of adolescent growth into a component due to the pubertal peak and one due to ongoing prepubertal velocity is evaluated.
Methods: Data for 120 boys and 112 girls are available from 4 weeks to adulthood. The curve-fitting method is kernel estimation for distance, velocity and acceleration.
Results: The age of peak velocity and the age of stopping of the PS are well determined. In contrast, the age of onset of the PS is less well determined. Intensity is less variable for the parameter peak velocity than for maximal acceleration. It is feasible to decompose adolescent growth into a component due to the pubertal peak and one due to ongoing prepubertal growth.
Conclusions: Nonparametric curve-fitting methods which do not rely on a parametric growth model can be successfully used to extract individual characteristics of the PS.

1. Introduction

In this paper we want to discuss how the pubertal growth spurt (PS) can be characterized meaningfully—both in biological and statistical terms—by a set of parameters. Some aspects of this study might be relevant for characterizing other curves which show changes over age or time, such as concentration of hormones.

The PS has mostly been studied for height but is present in most other anthropometric variables (Tanner 1962, Tanner, Whitehouse, Marubini *et al.* 1976, Gasser, Kneip, Binding *et al.* 1991a, Gasser, Kneip, Ziegler *et al.* 1991b). Visually, it appears as a velocity peak arising out of a steadily changing prepubertal velocity. From child to child the PS varies in its timing (age of occurrence), in its intensity and its duration (see §2 for an illustration). The timing can be characterized by the age of onset of the PS, by the age of peak velocity—the most popular measure of timing—and by a parameter characterizing the end of the PS (Gasser, Köhler, Müller *et al.* 1984a). From these timings a duration of the PS can be obtained as the difference between the end and the onset of the PS. Traditionally, the intensity of the PS has been defined to be peak velocity; we argue here that the velocity gained above the prepubertal level is a more adequate measure of intensity. The intensity could also be further subdivided into two aspects: the intensity of the increase in pubertal growth (the maximal acceleration) and the intensity of stopping the spurt (the maximal deceleration) (Gasser *et al.* 1984a). A prerequisite for determining such parameters is the possibility of estimating acceleration curves (Gasser and Müller 1984, Gasser, Müller and Mammitzsch 1985). We would like to discuss the usefulness of these definitions, and show the substantial random variability which has to be tolerated when determining individual parameters by curve fitting from a quite limited individual data set (usually yearly or half-yearly measurements).

Annals of Human Biology ISSN 0301–4460 print/ISSN 1464–5033 online
http://www.tandf.co.uk/journals
DOI: 10.1080/0301446001001302 5

The contribution or increment due to adolescent growth has usually been defined to be the growth occurring after the onset of the PS (Tanner *et al.* 1976). While this parameter is straightforward, it has the drawback of not disentangling at all the contributions due to continued growth roughly at the prepubertal level (as manifested in hypogonadal growth without a spurt) and the additional growth initiated by the onset of puberty. In Largo, Gasser, Prader *et al.* (1978) a relatively simple procedure has been suggested for separating these two increments, while in Stützle, Gasser, Molinari *et al.* (1980) a sophisticated modelling procedure allows a more appealing separation of continued prepubertal growth and additional pubertal growth. In Sheehy, Gasser, Molinari *et al.* (2000) a combination of these two definitions is used and this definition will be evaluated here.

2. Subjects and methods

2.1. Subjects and measurements

Based on the Zurich longitudinal growth study, 112 girls and 120 boys with essentially complete measurements were analysed. Visits were at 1, 3, 6, 9, 12, 18 and 24 months and then annually, but half-annually in pubertal years. For this methodological contribution we will focus on the variables height, leg height and biiliac width.

Biiliac width was measured with callipers, standing and sitting height with a Harpenden stadio-meter and leg height is obtained as the difference. Further details may be found in Gasser *et al.* (1991a, b) or Sheehy, Gasser, Molinari *et al.* (1999).

2.2. Statistical methods

Traditionally, the PS was individually quantified by parametric nonlinear growth models such as the logistic model (Tanner *et al.* 1976) or the Preece–Baines model (Preece and Baines 1978). A problem with these parametric models is the estimation bias (Gasser, Müller, Köhler *et al.* 1984b). An alternative is the use of nonparametric fitting methods such as splines (Largo *et al.* 1978) or kernel methods (Gasser *et al.* 1984a). Here, we will use the kernel method since it allows the estimation of distance, velocity and acceleration curves. The bandwidth (or smoothing parameter) is an important parameter governing how good the curve fit will be. The optimal bandwidth can be estimated from the data (Gasser, Kneip and Köhler 1991c, Brockmann, Gasser and Herrmann 1993) but the individual values are quite variable and it is advisable to use an average bandwidth. In a recent paper, Ramsay, Bock and Gasser (1995) considered the estimation of acceleration for height feasible but rather unreliable for anthropometric variables not as well determined as height. While our experience makes us feel a little more optimistic, it has to be conceded that in children with a variable growth pattern, some care is needed in curve fitting, possibly with some fine tuning of the data analysis.

From the individual velocity and the acceleration curves the following timings can be determined for the PS (see figure 2 for an illustration).

- age of onset of PS: age of minimum velocity before PS ($T6$)
- age of maximum intensity: age of maximal acceleration during the PS ($T7$)
- age of peak velocity: maximum velocity ($T8$)
- age of maximum stopping: given by maximum deceleration ($T9$)

From these timings, the duration of the PS is obtained as $T9 - T6$. It has to be noted that the determination of $T6$ is not always unambiguous, in particular for leg height.

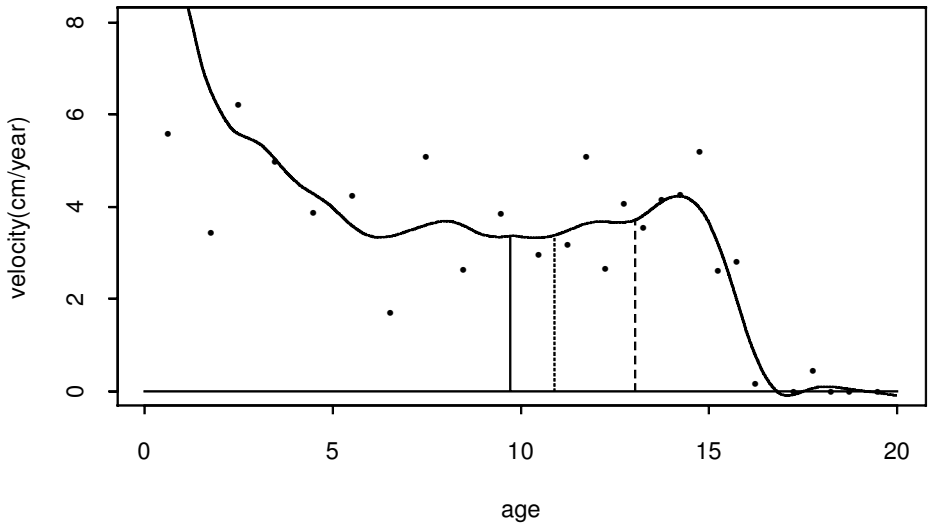

Figure 1. Velocity of leg height of a child (points = raw velocity, solid line = kernel estimated velocity). Vertical lines indicate potential candidates for the onset of the PS.

This is illustrated in figure 1 in the velocity curve of legs of a particular child. After a small mid-growth spurt at about age 7, the velocity slowly increases to reach peak velocity shortly before age 15. Formally, it is not quite clear where age of onset of the PS should be defined; a comparison with the velocity curves of other variables (e.g. sitting height or biiliac width) is often helpful in reaching a decision.

The intensity of the PS is characterized in a natural way by the maximal acceleration (i.e. the acceleration at $T7$), but this quantity is not always easy to determine. Traditionally, peak velocity (i.e. the velocity at $T8$) has been used as a measure of intensity of the PS. A disadvantage of this measure is that it is confounded with the size of the prepubertal velocity level which varies between individuals and between different anthropometric variables. A better measure of intensity would be the size of the velocity peak above the prepubertal level (i.e. peak velocity minus velocity at onset $T6$; Largo *et al.* 1978). The intensity of stopping the PS is given by the maximal deceleration (at $T9$). Due to the smoothing involved in nonparametric curve fitting the peak velocities and accelerations can be underestimated to some extent.

The adolescent gain—traditionally the growth from the onset of the PS till adulthood—should ideally be subdivided into a contribution due to the spurt and one due to the prepubertal velocity level. There is evidently some arbitrariness in any definition but common sense and experience from pathological conditions may eventually lead to a useful definition. The contribution due to the PS was defined by Largo *et al.* (1978) to be the velocity peak above a constant line given by the velocity at onset $T6$ (see figure 2). This then leads to a small component after T^* representing 'late growth', which is not fully satisfactory. Stützle *et al.* (1980) proposed a sophisticated modelling procedure with a prepubertal and a pubertal component. The two components enter the model not additively, but there is an interaction term by which the onset of the PS begins to switch off prepubertal growth smoothly (it is down to zero at age of peak velocity $T8$). Such a switch-off is in line with the fact that pubertal hormones do not only initiate rapid growth via a spurt but that they also stop further

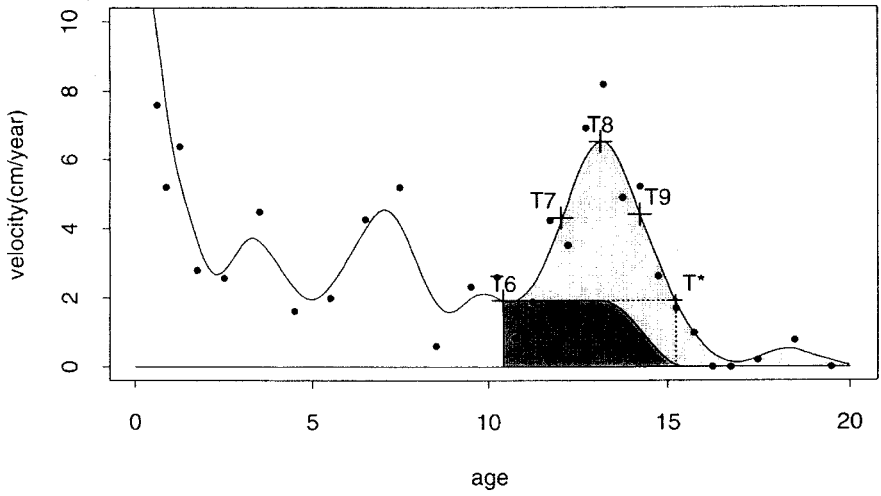

Figure 2. Illustration of the decomposition of the adolescent gain into a contribution due to level (dark grey area) and one due to spurt (light grey area). Definition of Largo *et al.* (1978) is delineated by dashed line (constant until T^*, 'aftergrowth' after T^*).

growth by calcification. That prepubertal growth continues in some way into adolescence is made plausible by hypogonadal growth which shows a slowly decaying velocity trend without PS, continuing well beyond the age of normal puberty.

Our definition (Sheehy *et al.* 2000) is in the spirit of Stützle *et al.* (1980) with some notable differences (figure 2): firstly, we do not need a sophisticated modelling procedure but define a switch-off function directly for the velocity curves obtained by kernel estimators. Secondly, prepubertal velocity is assumed to stay constant from $T6$ to $T8$ and declines after $T8$. This makes more sense to us since at $T8$ the acceleration becomes negative. The exact definition of the switch-off function is as follows:

$$\text{switch}(x) = VT6/2\,\{\cos\,(\pi(x - T8)/(T_{\max} - T8)) + 1\}$$

where x = age, $VT6$ = velocity at take-off $T6$, and $T_{\max}$ = maximum of ($T9$, $T_{97.5}$), where $T_{97.5}$ is the age where 97.5% of adult size are reached. This allows then the definition of a contribution due to the spurt and one due to the level in the adolescent period.

We will check how well kernel estimation of timings, duration and intensities work by statistical simulation: the structural average distance curves (Gasser *et al.* 1991a, b) are assumed to be true growth curves. From these true curves we obtain pseudo-measurements at the same ages as for the real data by adding a normal random variate with a variance seen in our sample (which, of course, depends on age, see Gasser, Sroka and Jennen-Steinmetz 1986). The PS parameters are then extracted from the pseudo-measurements by kernel estimators. This procedure is repeated 400 times to determine the variance and the bias of our estimation procedure. This error variance can then be compared with the variance seen in the sample, which is composed of biological variance plus error variance. Let us exemplify the statistical principle behind this for the age of take-off $T6$. Let $T6_i$ denote the true age of take-off for child no i, and $\hat{T}6_i$ the age of take-off actually determined by curve fitting. They are related as follows:

$$\hat{T}6_i = T6_i + E_i$$

where E_i = error due to curve fitting.

Since the error due to curve fitting E_i can be assumed to be independent of the unknown true value $T6_i$, the following relation holds for the variances:

$$\mathrm{Var}(\hat{T}6_i) = \mathrm{Var}(T6_i) + \mathrm{Var}(E_i)$$

Ideally, the error variance $\mathrm{Var}(E_i)$ should be small compared to the observed variance $\mathrm{Var}(\hat{T}6_i)$.

3. Results

Timings: As shown in table 1, the error variances arising from kernel estimation—obtained by simulation—are relatively small compared to the sample variances, and they are smaller for boys than for girls. This is plausible, since the location of a larger peak is easier to quantify. As expected, the age of onset of the PS ($T6$) is not as well determined as other timings, in particular age of peak velocity ($T8$). The age of peak acceleration ($T7$) is somewhat better determined statistically than $T6$, despite the involvement of the acceleration curve. The age of peak velocity ($T8$) is well determined, the error variance is about 6% of the sample variance. Surprisingly, the age of peak deceleration ($T9$), roughly equivalent to the total growth period is well determined and for legs, it is the most accurate measure. This has to do with the fact that the PS for legs is not so strong—making $T8$ more difficult to estimate—whereas the stopping is quite rapid (Gasser *et al.* 1991a). There is no discernible systematic error (or bias) when determining the timings.

Durations: As a consequence of the results for timings, durations can also be determined accurately enough. Inevitably, the involvement of two timings increases the variance of the statistical error.

Intensities: While peak velocity is statistically well determined (i.e. has a relatively low error variance), it is biologically less meaningful as a measure of intensity of the PS compared to maximum acceleration or peak size (i.e. the size of the velocity peak above the prepubertal level).

Table 1. Variances observed for $n = 120$ boys and $n = 112$ girls for PS timings, compared with error variance (400 simulations; units: year^2).

		Boys			Girls		
Parameter	Variance	Standing height	Leg height	Biiliac width	Standing height	Leg height	Biiliac width
$T6$	observed	1.19	1.37	1.93	0.98	1.44	0.69
	simulated	0.10	0.27	0.44	0.25	0.42	0.31
$T7$	observed	0.88	1.02	1.32	1.02	1.35	0.72
	simulated	0.17	0.16	0.37	0.26	0.31	0.28
$T8$	observed	0.85	0.85	0.96	1.02	1.12	0.92
	simulated	0.03	0.05	0.05	0.07	0.22	0.05
$T9$	observed	0.85	1.08	1.02	1.06	1.02	1.08
	simulated	0.05	0.03	0.12	0.11	0.06	0.10

As shown in table 2, peak size is well determined statistically since the error variance is a fraction of the sample variance. This fraction is somewhat higher for girls, compared to boys, except for biiliac width. As to be expected the bias is negative so that the kernel estimated peak size is smaller than the true one. However, in quantitative terms, bias is small compared to error variance. Therefore, bias is not tabulated. Maximal acceleration (table 2) shows a relatively higher error variance, compared to peak size, and its bias is also more appreciable.

We have applied our new definition for estimating separately the adolescent contribution due to the spurt and one due to the level to the real data set and also the definition used by Largo *et al.* (1978). Table 3 shows that the correlations for the two definitions 'contribution due to the spurt' are high so that the details of the definition do not matter too much. As argued before, the new one is somewhat more appealing in biological terms and does not necessitate a third adolescent component of 'late growth' as does the old definition.

Table 4 shows that the contribution due to the spurt is quite well determined while the contribution due to the level shows a relatively high error variance in comparison with the observed variance.

Table 2. Variances observed for $n = 120$ boys and $n = 112$ girls for pubertal peak velocity and for maximal acceleration compared with error variance (400 simulations; units: $(\text{cm/year})^2$ and $(\text{cm/year}^2)^2$).

		Boys			Girls		
Parameter	Variance	Standing height	Leg height	Biiliac width	Standing height	Leg height	Biiliac width
Peak size	observed	0.822	0.329	0.046	0.696	0.314	0.078
	simulated	0.159	0.052	0.007	0.161	0.08	0.010
Maximal acceleration	observed	0.466	0.108	0.017	0.314	0.094	0.034
	simulated	0.128	0.026	0.004	0.123	0.034	0.005

Table 3. Correlations between new and old definition of the contribution due to the PS.

Sex	Standing height	Leg height	Arm length	Sitting height	Biiliac width	Bihumeral width
Male	0.95	0.94	0.97	0.94	0.96	0.98
Female	0.92	0.85	0.90	0.94	0.95	0.96

Table 4. Variances observed for $n = 120$ boys and $n = 112$ girls for contributions due to the spurt and due to the level, compared with error variance (400 simulations; units: cm^2).

		Boys			Girls		
Parameter	Variance	Standing height	Leg height	Biiliac width	Standing height	Leg height	Biiliac width
Spurt	observed	6.65	2.50	0.56	5.95	3.49	0.91
	simulated	1.25	0.54	0.12	1.72	0.93	0.14
Level	observed	11.77	6.41	0.41	11.33	5.82	0.67
	simulated	3.26	2.26	0.16	6.27	2.93	0.21

4. Discussion

The timings characterizing the age of occurrence of the PS can be determined statistically with a varying accuracy: while the age of onset of the PS shows a relatively large error variance, the age of peak velocity is very well determined indeed. This can be attributed to the fact that in many children velocity may start as a slow increase in late childhood so that the onset is not obvious visually and in statistical terms. On the other hand, the age of maximal deceleration is well determined, which is a positive surprise given the fact that the acceleration curve is rather difficult to determine reliably. The age of maximal deceleration is of biological interest since it corresponds roughly to the total growth period. As to be expected from statistical arguments, bias is not a problem when estimating these timings.

We studied two parameters to characterize the intensity: the height of the pubertal velocity peak above the prepubertal level and maximal acceleration during the PS. While peak size is statistically well determined, error variance is a sizeable portion of sample variance for maximal acceleration. This is not too surprising given the difficulty of extracting an acceleration curve from a limited data set (typically about 32 measurements per child), which can also show considerable random variability. As to be expected from statistical theory, we see a negative bias—indicating an underestimation of intensity—and more so for maximal acceleration than for peak size. Here, and for other parameters, the error variance is in general more important for girls than for boys: this can be explained by the fact that girls have a smaller PS, and a smaller PS is more difficult to determine than a large one from noisy data (intuitively and by statistical arguments). Biiliac width is usually an exception and this can be explained in the same way: the PS is as large for girls as for boys. As a consequence, the size of the velocity peak above prepubertal level might usually be a better measure of intensity of the PS compared to maximal acceleration.

We have already argued in favour of subdividing the growth increment in adolescence into two increments (Sheehy *et al.* 2000): one to be attributed to the appearance of the PS and one constituting ongoing growth at the prepubertal level. The contribution due to the spurt is statistically reasonably well determined, due to a relatively low error variance. The increment due to the level, on the other hand, shows a proportionally large error variance. This can be attributed to the difficulty in determining reliably the onset of the PS, which is crucial in the definition of the level (see timings, above).

Acknowledgement

This work was supported by the Swiss National Science Foundation (Project no.3200-045829.95/2 and a Marie Heim-Voegtlin Fellowship).

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Zusammenfassung. *Hintergrund:* Dieses ist eine methodische Untersuchung zum Problem der Schätzparameter des puberalen Wachstumsspurts (PS). Die Variabilität, die bei der Bestimmung des Zeitpunktes, der Intensität und Dauer des PS bei der Körperhöhe, der Beinhöhe und bei der Beckenbreite auftritt, wird über eine realistische Simulation geschätzt. Weiterhin wird das Wachstum in der Adoleszens hinsichtlich des puberalen Wachstumsgipfels und der fortwährenden prepuberalen Geschwindigkeit ausgewertet.
Methoden: Für 120 Jungen und 112 Mädchen liegen Daten von einem Alter von 4 Wochen bis zum Erwachsenenalter vor. Als Kurven-Glättungsmethode wurde die Kernel-Schätzung für Zunahme, Geschwindigkeit und Beschleunigung genutzt.
Resultate: Das Alter bei der maximalen Wachstumsgeschwindigkeit und das Alter beim Abschluß des PS sind gut feststellbar. Demgegenüber ist das Alter des Einsetzens des PS weniger gut feststellbar. Die Intensität der maximalen Wachstumsgeschwindigkeit ist weniger variabel als die Intensität der maximalen Beschleunigung. Es ist möglich das Wachstum in der Adoleszens in einen Bereich des puberalen Wachstumsgipfel und ein fortwährendes prepuberales Wachstums zu unterteilen.
Schlußfolgerungen: Nichtparametrische Kurvenglättungsmethoden, welche nicht auf einem parametrischen Wachstumsmodell beruhen, können erfolgreich verwendet werden, um individuelle Charakteristiken des PS herauszuarbeiten.

Résumé. *Arrière plan*: ceci est une analyse méthodique du problème de l'estimation des paramètres de la poussée pubertaire (PP). La variabilité de détermination de la chronologie, de l'intensité et de la durée de la PP pour la stature, la hauteur de la jambe et la largeur biiliaque, est estimée au moyen d'une simulation réaliste, puis est effectuée une décomposition de la poussée de l'adolescence en un composant du au pic pubertaire et en un autre du à la continuation de la vélocité prépubertaire.
Méthodes: des données sur 120 garçons et sur 112 filles sont disponibles de l'âge de quatre semaines jusqu'à l'état adulte. La méthode d'ajustement des courbes est celle de l'estimation de fond (kernel), pour la distance, la vélocité et l'accélération.
Résultats: l'âge de la vélocité de pic et celui de l'arrêt de la PP sont bien déterminés. Par contre, l'âge au début de la PP est moins précis. L'intensité est moins variable pour la vélocité de pic que pour l'accélération maximale. Il est possible de décomposer la poussée de l'adolescence en un composant du au pic pubertaire et un autre du à la poursuite de la croissance prépubertaire.
Conclusions: les méthodes non paramétriques d'ajustement des courbes, qui ne reposent pas sur un modèle paramétrique de croissance peuvent être utilisées avec succès pour extraire des caractéristiques individuelles de la PP.